RESEARCH

Hierarchical ranking of the Dow Jones index using the ELECTRE-III method

Ranking jerárquico del índice Dow Jones usando el método ELECTRE-III

Date received: October 1st, 2021

Eva Luz Miranda Espinoza¹, Manuel Muñoz Palma² and Pavel Anselmo Álvarez Carrillo³

Date approved: December 15th, 2021

- ⁴ Business Consultant, Master in Financial Management from Universidad TecMilenio, certified Public Accountant from Universidad de Sonora. Member of the UADEO research group. Her research interests include fuzzy logic in the field of finance and fuzzy systems. https://orcid.org/0000-0002-0992-6956, Email: mirandaeeva@hotmail.com, tel: (662) 127 8478
- ² Corresponding author. PhD in Administrative Sciences from Universidad de Occidente, enrolled in Portal del Consejo Nacional de Ciencia y Tecnologia (PNPC) by SEP-CONACyT. Full-time research professor in the Department of Management at Universidad de Sonora. Research areas: Finance, fuzzy logic, and intelligent models in Finance. Currently a member of the National System of Researchers (Level C). Institutional email: manuel.munoz@unison.mx , Blvd. Luis Encinas y Rosales S/N, Col. Centro Hermosillo, Sonora, Mexico . https://orcid.org/orcid-search/sear=0000-0003-2980-4814 . Tel (662)181 0280
- ³ Full-time professor at Universidad Autonoma de Occidente. Currently, is a Level I of the National System of Researchers (SNI). Pavel Anselmo earned his PhD in Administrative Sciences from Universidad de Occidente, a program recognized in the National Register of Quality Graduate Programs (PNPC) by SEP-CONACyT. His research interests include web data collection, multicriteria analysis, and decision support system.

Email: pavel.alvarez@uadeo.mx. Blvd. Lola Beltran, C.P. 80020 Culiacan Rosales, Sinaloa, Mexico, http://orcid.org/0000-0003-4445-076X . Tel: (667) 759 1300

Abstract

The objective of the article is to present a multicriteria hierarchical process (MCHP) approach to decision making in the selection of stocks of the main companies of the Dow Jones index. One of the problems that investors often face is deciding which stocks should be included in an investment portfolio. The article allows investors to answer this question, through an MCHP approach and the ELECTRE III method using different criteria based on the financial relationships of profitability, liquidity, market, and efficiency. In this process, the investor generates a global ranking and a ranking of each subgroup of criteria regarding the investor's preferences.

Keywords: Hierarchical multicriteria process, ELECTRE III, Financial ratios, Dow Jones.

JEL code: C61, M40, G15

Resumen

El objetivo del artículo es presentar un enfoque de proceso jerárquico multicriterio para la toma de decisiones en la selección de acciones de las principales empresas que cotizan en el índice Dow Jones. Uno de los problemas que suelen enfrentar los inversores es decidir qué acciones deben incluirse en un portafolio de inversión. El artículo permite a los inversores dar respuesta a esa pregunta, mediante un enfoque jerárquico y el método ELECTRE III utilizando diferentes criterios basados en las ratios financieras de rentabilidad, liquidez, mercado y eficiencia. En este proceso el inversor genera un ordenamiento a un nivel global y un ordenamiento subgrupo de criterios considerando las en preferencias del inversor.

Palabras clave: Proceso jerárquico multicriterio, ELECTRE III, Ratios financieras, Dow Jones.

Código JEL: C61, M40, G15

1. Introduction

The evolution of financial theory has enabled the conceptualization of financial management from various perspectives. Its importance becomes evident when facing dilemmas such as leverage versus profitability, always seeking the timely provision of resources to support effective decision-making and ensure financial returns that drive business growth.

One of the main challenges in operating within the stock market lies in risk management. In this context, concepts such as hedging, insurance, and diversification become highly relevant. Bodie and Merton (2003) argue that diversification, by distributing capital among multiple risky assets, reduces exposure to individual asset risk. Likewise, Merton's dynamic continuous-time hedging technique serves as a bridge between Kenneth Arrow's theoretical model of complete markets and the practical needs of personal financial planning in real-world contexts (Bodie, 2019). In this sense, the investment portfolio, grounded in classical financial theory, seeks to optimize the risk-return trade-off through diversification.

Traditional models that consider only return and risk criteria-without accounting for investor preferences-may propose portfolios that do not reflect the investor's interests. In contrast, models that do incorporate such preferences, along with additional decision-making criteria, achieve a more appropriate alignment with the investor profile (Ehrgott et al., 2004). Diversification is also closely related to risk behavior according to each investor's profile (Basilio et al., 2018). However, conventional tools often fail to consider increasingly complex and multifactorial scenarios-economic, social, environmental—that involve multiple and conflicting criteria (Guerrero-Baena, Gómez-Limón, & Fruet-Cardozo, 2014).

This research adopts both quantitative and qualitative approaches and focuses on the factors influencing decision-making in the development of investment portfolio selection strategies in the context of the COVID-19 pandemic and its effects on financial ratios of companies listed on the New York Stock Exchange. Although the importance of investment portfolio selection has been addressed in various studies, current approaches often overlook investor profiles and the existence of conflicting criteria. In this regard, analytical tools are needed to meet new demands in decision-making processes

This study addresses portfolio selection as a multicriteria ranking problem through the adaptation of the hierarchical multicriteria process proposed by Corrente et al. (2012), based on the natural hierarchy that characterizes the criteria involved in stock selection. The portfolio selection problem inherently presents a hierarchical structure of criteria. For this analysis, the ranking of stocks considers seven macro-criteria (groups of criteria): market ratios, operating results, market value ratios, financial and economic profitability, liquidity, efficiency, and dividends. The objective of the study is to generate a hierarchical ranking of companies listed on the Dow Jones Index. This entails organizing stocks by groups of criteria to analyze their performance within each group, enabling the explanation of stock behavior and investment potential.

The structure of this paper is as follows: Section 2 provides a literature review. Section 3 outlines the methodology of the hierarchical multicriteria process, incorporating the hierarchical version of the ELECTRE III method. Section 4 presents the performance analysis of the companies' stocks and the corresponding results. Section 5 contains the conclusions.

2. Literature Review

The New York Stock Exchange (NYSE) was established in 1790. The Dow Jones Industrial Average represents the top 30 industrial stocks traded on the NYSE. These companies can significantly influence overall market movements, as the index serves as a robust indicator of the U.S. economy and investor confidence in specific securities. As a global leader, the NYSE serves as a venue for investors seeking access to capital and participation in global markets. Its unique model helps minimize execution risk and stock price volatility. Chahuán (2018) noted a positive correlation between the Dow Jones Index and other markets, such as Chile's, where the index correlates more strongly with revenue than



with business outcomes. Decision-makers play a critical role in optimizing returns and minimizing investor risk when constructing a portfolio. Useche (2015) emphasized the contribution of financial institutions in providing more accurate advisory processes that cater to the personal expectations and specific interests of investor clients.

Risk, as analyzed by various authors, has a direct effect on corporate financing decisions, since the composition of a firm's capital structure and its level of financial leverage or debt ratio directly influence firm value. Milanesi (2016), in studies conducted on the Argentine stock market to evaluate the effect of volatility at varying debt levels, confirmed the consistency of the proposed model linking volatility, value, and probability of financial failure. An increase in external capital raises insolvency risk, which is reflected in a decline in stock value. López-Dumrauf (2003) argued that firms must strike the right financing mix to minimize capital costs and maximize firm value. Elselmy, Ghoneim, and Elkhodary (2019) highlighted the importance of accounting information in financial statements to identify the indicators needed for constructing business models for portfolio integration in the Egyptian stock market. Mansour et al. (2019) proposed a possibility theory and a model that allows for trade-offs between investor preferences regarding multiple incommensurable objectives in uncertain environments.

In portfolio selection under the principles of corporate social responsibility and the use of multiobjective and multi-criteria techniques, Suárez, Pimiento, and Duarte (2018) noted that such tools support socially responsible investors in identifying portfolios that meet their goals of maximizing returns while minimizing risks. Cervelló, Guijarro, and Michniuk (2014) reported a positive riskadjusted return for the flag pattern based on Dow Jones intraday data over a time horizon of over 13 years. Ariza and Cadena (2017) applied mixed beta to assess asset risk or predict returns, which aided in capital budgeting, asset valuation, equity cost estimation, and explaining risk in the context of interest rates.

A wide range of intelligent systems has been proposed to solve the portfolio selection problem,

such as reinforcement learning (Moody et al., 1998; Moody & Saffell, 2001; OJ. et al., 2002), neural networks (Kimoto et al., 1993; Dempster et al., 2001), genetic algorithms (Mahfoud & Mani, 1996; Allen & Karjalainen, 1999; Mandziuk & Jaruszewicz, 2011), decision trees (Tsang et al., 2004), support vector machines (Tay & Cao, 2002; Cao & Tay, 2003; Lu et al., 2009), and expert boosting and weighting (Creamer & Freund, 2007; Creamer, 2012). Although these studies attempt to interpret market conditions and predict future trends, such techniques are often unsuitable for small investors due to the required level of expertise. Moreover, they do not facilitate comparisons across multiple ambiguous criteria (Boonjing & Boongasame, 2016).

This study presents a multi-objective approach involving fuzzy parameters, where possibility distributions are represented by fuzzy numbers, and investor preferences are explicitly incorporated using satisfaction functions. Aldalou and Perçin (2018) proposed a financial performance evaluation model. Fuzzy AHP was used to assign weights to evaluation criteria, while Fuzzy TOPSIS ranked the alternatives. The model was applied to listed airline companies on the Istanbul Stock Exchange for the period 2012-2016. A portfolio optimization model based on Markowitz's classical mean-variance model was proposed by Ehrgott et al. (2004) and applied to the Standard & Poor's database of 1,108 mutual funds. Sánchez, Milanesi, and Rivitti (2010) studied portfolio problems using AHP on four Argentine firms and evaluated their performance through five types of financial ratios (profitability, activity, liquidity, solvency, and market value) calculated from accounting data since 2006. Mohammad et al. (2012) applied the TOPSIS method to a sample of 18 top companies from different industries listed on the Tehran Stock Exchange (TSE) over five years.

Bahloul and Abid (2013) developed combined AHP and Goal Programming (GP) methods to study the impact of investment barriers on international portfolio selection. AHP was used to identify suitable international equity portfolios based on investment barriers, while GP incorporated market weights for maximum return, minimum variance, and AHP portfolios to determine optimal international equity portfolios. Pätäri et al. (2017) compared the effectiveness of scale median (SM), TOPSIS, AHP, and DEA in identifying future topperforming stocks in U.S. equity samples.

Altinirmak et al. (2016) applied AHP-PROMETHEE to assess the performance of nine investment trusts listed on BIST (Turkey's stock exchange). Albadvi, Chaharsooghi, and Esfahanipour (2006) noted the application of PROMETHEE to the Tehran Stock Exchange using surveys, financial reports, and expert opinions to evaluate criteria and organizations. Basilio et al. (2018) used principal component analysis and the PROMETHEE II method to compare financial performance indicators across stocks traded on the São Paulo Stock Exchange.

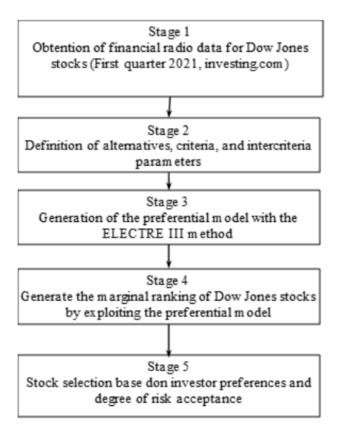
Lima and Soares (2013) applied the ELECTRE III method to select assets for a buy-and-hold strategy and to test whether the chosen assets outperformed the market as measured by the Portuguese Market (PSI-20TR). Vezmelai, Lashgari, Index and Keyghobadi (2015) used ELECTRE III to rank 20 companies listed on the Tehran Stock Exchange in 2011 and compared the results with the TSE's own rankings. Boonjing and Boongasame (2016) proposed a combinatorial portfolio selection using ELECTRE III to support small investors in making investment decisions. Xidonas et al. (2009) applied ELECTRE III to classify companies into eight sectors or industries as part of a Pareto investment portfolio. Multicriteria decision-aid (MCDA) methods have been widely used to address portfolio selection problems. The ELECTRE III method, in particular, has been employed within the MCDA framework for finance and portfolio selection problems (Spronk et al., 2016; Govindan & Jepsen, 2016).

3. Methodology

One of the basic features of multicriteria analysis is the comparison of alternatives based on a series of criteria. Therefore, multicriteria ranking methods are designed to construct a recommendation on a set of alternatives according to the preferences of the expert or decision-maker.

To generate the ranking of the main stocks, the hierarchical multicriteria process is applied to the stocks listed on the New York Stock Exchange that are part of the Dow Jones Index, considering financial ratios. The data for these stocks corresponds to the first quarter of 2021 and can be found on the financial portal www.investing.com.

Figure 1. Research model for the marginal ranking of the 30 Dow Jones stocks.



Source: Own elaboration.

Figure 1 presents the working framework of this research, which is defined in five stages. Stage 1 identifies the main data from the financial ratios of the 30 Dow Jones companies. Stage 2 corresponds to an intelligence phase in decision-making; here, the decision criteria representing the stocks must be defined, as well as the decision alternatives (the companies listed on the stock exchange), and the parameters of the multicriteria method (ELECTRE III). In Stage 3, a multicriteria analysis method is applied-in this case, the ELECTRE III method is used to generate a preference model (a valued matrix of the stocks). Stage 4 corresponds to the exploitation of the preference model; in this step, a distillation process is used to rank the stocks. In Stage 5, the ranking results and information analysis are presented to the investor for the final selection



of stocks. In this regard, the process and method consider the investor's profile and the level of risk they are willing to accept.

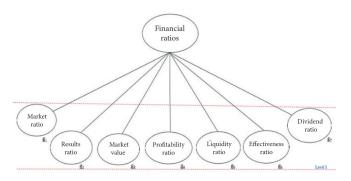
The following section describes the hierarchical multicriteria process and the ELECTRE III multicriteria method used to rank the stocks.

3.1 Hierarchical Multicriteria Process

In the MCDA process, a set of alternatives is defined as $A = \{a_1, a_2, \ldots, a_m\}$, along with a coherent family of criteria $G = \{g_1, g_2, \dots, g_m\}$. Any MCDA method develops an overall preference method as an aggregation procedure. The method generates a recommendation in the form of a ranking of alternatives in descending order from best to worst. The first stage of the portfolio selection problem involves generating a stock evaluation ranking. For this problem, it is easy to observe the hierarchical structure of the decision criteria. Therefore, it is common for a practical application to impose a hierarchical structure (Corrente et al., 2012). For this reason, the multicriteria ranking of stocks is generated using a new method: the Multiple Criteria Hierarchy Process (MCHP).

A traditional multicriteria analysis method evaluates the stocks at the same level, assessing all the criteria at once (see Figure 2). In this way, one can identify which stocks are the best and which are the worst, but it is not possible to understand how some subcriteria (subgroups of ratios)—such as market, performance, or liquidity ratios—interact in the evaluation of a stock and influence stock selection. In this sense, a different method would be valuable to assess the stocks by subsets of criteria at different levels, following the MCHP methodology to solve the stock selection problem.

It often happens that a practical application imposes a hierarchical structure of criteria (Salvatore Corrente et al., 2012). In the stock selection problem, a large number of decision criteria are involved. In fact, evaluating stock selection requires various types of information, commonly addressed using the Dow Jones indices. Considering these characteristics, the MCHP approach allows the stock selection problem to be broken down into subproblems by using a criteria hierarchy to facilitate a deeper analysis. **Figure 2.** Evaluation criteria at the same level for the stock selection problem.



Source: Own elaboration.

To address decision-making problems in which evaluation criteria are considered at the same level, a hierarchical structure is instead used to organize them within a specific segment of the problem. The basic idea of the Multiple Criteria Hierarchy Process (MCHP) is based on considering preference relations at each node of the hierarchical tree of criteria. These preference relations refer both to the phase of eliciting preference information and to the phase of analyzing a final recommendation by the decision-maker (Corrente et al., 2012).

A hierarchical structure of criteria can be seen as a criteria tree. The tree structure is of particular interest to the expert or decision-maker and clusters a subset of criteria into leaves. These leaves decompose the overall problem into smaller problems, allowing for a better understanding of the interaction among elementary criteria. Figure 2 addresses a multicriteria decision-aid problem in which all criteria are evaluated at the same level. However, the same problem can be analyzed as smaller subproblems through a hierarchical structure. In the tree structure of criteria, some leaves contain branches with additional leaves, forming a tree of subproblems. Corrente, Figueira, Greco, and Słowiński (2017) integrate the MCHP with the ELECTRE III method. To explain the ELECTRE III hierarchy, we follow the notation of Angilella et al. (2018):

G is the comprehensive set of all criteria at all levels considered in the hierarchy.

 G_0 is the root of the criteria.

 l_G is the set of indices of the criteria in G.

 $E_G \subseteq l_G$ is the set of indices of the elementary criteria.

g is the generic criterion (where r is a vector whose length corresponds to the criterion's level).

 $g_{(r,1)}, \ldots, g_{(r,n(r))}$ are the immediate subcriteria of criterion g_r (located one level below g_r).

 $E(g_r)$ is the subset of indices of all elementary criteria descending from g_r .

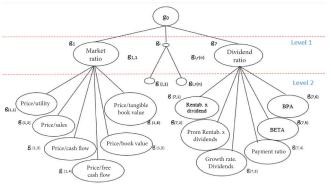
E(F) is the set of indices of the elementary criteria desceding from at least one criterion in the subfamily $F \subseteq G$ (que es, $E(F) = U_{g_r \in E}(g_r)$.

 G_r is the set of subcriteria of g_r located at level l in the hierarchy (below g_r).

To better understand the notation above, in a hierarchical structure, Level 1 contains the macro-criteria, and the elementary criteria descending from these macro-criteria decompose the subproblem. The complete set of elementary criteria is contained within E_G . A different approach can be implemented for the multicriteria decisionaid problem by generating a hierarchical structure with respect to the criteria of interest at a particular level of the hierarchy.

The problem of stock selection for portfolio integration can be addressed as a hierarchical problem, where some macro-criteria may encompass elementary criteria from a deeper level in the hierarchy. Figure 3 illustrates a summarized structure (two macro-criteria) of the complete hierarchical problem of stock selection within the Dow Jones index. The macro-criterion Market Ratio (g_1) integrates six Elementary criteria, Results ratio (g₂) integrates 8 elementary criteria, and so on, up to the Dividend ratio macro-criterion (g_7) which integrates six Elementary criteria. The evaluation of Dow Jones index stocks includes 47 elementary criteria structured in a two-level hierarchy: Level 1 defines seven macro-criteria (non-elementary), and Level 2 contains 47 elementary criteria that constitute the macro-criteria of Level 1.

Figure 3. Simplified MCHP structure for NYSE stock selection.



Source: Own elaboration.

3.2 Hierarchical ELECTRE III Method

The adapted version of the ELECTRE III hierarchy was first introduced by Corrente et al. (2017). The ELECTRE method is developed in two steps. The first step involves the aggregation of preferences, where informati on is processed by constructing a model based on the valued outranking relation. This process is explained with an illustrative example in the Appendix. In the second step, the valued outranking relation is exploited through the distillation process, generating either a partial or complete ranking of alternatives. For each elementary criterion $g_t \in E_g$.

The elementary concordance index for each elementary criterion g_t is given by

$$\varphi_{t}^{i}(a,b) = \begin{cases} 1 & if \quad g(b) - g(a) \leq q_{t}(a,\xi b) \\ \frac{p_{t} - [g(b) - g(a)]}{p_{t} - q_{t}} & if \quad q_{t} < g(b) - g(a) < p_{t}(b,Qa) \\ 0 & if \quad g(b) - g(a) \geq p_{t}(b,Qa) \end{cases}$$

The elementary discordance index for each elementary criterion g_t is given by

$$d_{t}(a,b) = \begin{cases} 1, & \text{if} & g_{t}(b) - g_{t}(a) \geq v_{t}, \\ \frac{[g_{t}(b) - g_{t}(a)] - p_{t}}{v_{t} - p_{t}} & \text{if} & p_{t} < g_{t}(b) - g_{t}(a) < v_{t}, \\ 0, & \text{if} & g_{t}(b) - g_{t}(a) \leq p_{t}. \end{cases}$$

The partial concordance index for each nonelementary criterion g_t

$$C_r(a,b) = rac{\sum_{t \in E(g_r)} w_t arphi_t(a,b)}{\sum_{t \in E(g_r)} w_t}$$



Partial credibility index

$$\sigma_r(a,b) = \begin{cases} C(a,b)x \prod_{g \in E(g_r)} \frac{1 - d_r(a,b)}{1 - C_r(a,b)} & \text{if } d_r(a,b) > C_r(a,b) \\ C(a,b) & \end{cases}$$

The valued outranking relation generated in the previous step corresponds to the decision maker's preference model. The distillation method is used to exploit the preference model. Distillation is performed both in descending and ascending manners; consequently, the final preorder is obtained as the intersection of the two distillations. An overview of the distillation method is described in Giannoulis & Ishizaka (2010).

For the pair $a, b \in A$ in the hierarchical process, the alternatives are ordered in a partial or complete preorder for each non-elementary criterion g_r as follows:

 $aP_rb:a$ is strictly preferred to b on the macrocriterion g_r in at least one of the orderings, a is ranked before b, and in other ordering, a is at least as good as b.

 $aI_{p}b:a$ is indifferent to b on the macro-criterion g_{r} if both actions occupy the same position in the two preorders.

 $aR_{r}b:a$ is incomparable to b on the macro-criterion g_{r} if a is ranked better than b in the ascending distillation and b is ranked better than a in the descending distillation, or viceversa.

4. Analysis of Dow Jones stocks using the hierarchical multicriteria process

The analysis is based on the financial statements from the first quarter of 2021, obtained from the financial portal Investing and collected from the NYSE, which generates a performance index reporting on Dow Jones Index companies and indicating existing capabilities for investors (see Appendix, Table A.2.1). Financial ratios are used to select the macro-criteria to evaluate each company's performance (see Table 2). These provide insights into the company's financial situation and performance prospects, as well as an evaluation of a company's position relative to others.

The data obtained from the NYSE is grouped into seven dimensions used to evaluate the stocks listed on the Dow Jones Index. Each dimension consists of a subgroup of different indicators (elementary criteria). In total, there are 47 indicators to evaluate the stocks of the 30 companies in the Dow Jones Index. The NYSE data is used in this study with a new approach—the Hierarchical Multicriteria Process (MCHP)—to analyze stock performance, considering the interaction of subgroups of criteria at different levels within a hierarchy through the ranking of Dow Jones companies, as shown in Table 1. The macro-criteria for the stock selection problem, elementary criteria, and their corresponding weights are shown in Table 2.

Table 1. Dow Jones Index Companies.

Label	Company	Label	Company
Aı	3M	A16	Merck
A2	American Express	A17	Microsoft
A3	AT&T	A18	Nike
A4	Caterpillar, Inc.	A19	Pfizer
A5	Chevron Corporation	A20	Boeing
A6	Cisco	A21	Home Depot
A7	The Coca-Cola Company	A22	Procter & Gamble
A8	Dupont	A23	The Travelers Companies
A9	Exxon Mobil	A24	Walt Disney
A10	Goldman Sachs	A25	United Health Group
A11	Intel	A26	Raytheon Technologies
A12	IBM	A27	Verizon Communications
A13	Johnson & Johnson	A28	Visa
A14	JP Morgan Chase	A29	Wal-Mart
A15	McDonald's	A30	Walgreens Boots Alliance Inc.

Source: Own elaboration with data from NYSE.

Regarding the methodology proposed in Section 3.1, the HMCA (Hierarchical Multi-Criteria Analysis) is applied to solve the problem of stock selection for the construction of an investment portfolio. In the first step, the problem is structured into a multicriteria hierarchy, breaking it down into seven macrocriteria as subproblems of the stocks. As shown in the hierarchical structure in Figure 3, the stocks listed on the NYSE are organized in a hierarchy based on the seven macro-criteria and the 47 elementary criteria. The new hierarchical structure for the stock performance problem enables the analysis to align with HMCA. The approach implemented in this article evaluates each macro-criterion, allowing for analysis of the interaction between directly related, immediate sub-criteria. This is carried out by generating preferential models and rankings for each macro-criterion to understand how one stock performs relative to another, and how it influences the overall stock selection problem.

Table 2. Macro-criteria and elementary criteria for stock selection.

Index	Macro-criterion	Index	Elementary criteria	Weights
g1	Market ratios	g1,1	Price/earnings TTM ratio	0.0300
		g1,2	Price/sales TTM	0.0200
		g1,3	Price/cash flow MRQ	0.0100
		g1,4	Price/free cash flow TTM	0.0200
		g1,5	Price/book value MRQ	0.0400
		g1,6	Price/tangible book value MRQ	0.0300
g2	Results ratio	g2,1	Gross margin TTM	0.0200
		g2,2	Gross margin 5YA	0.0200
		g2,3	Operating margin TTM	0.0150
		g2,4	Operating margin 5YA	0.0150
		g2,5	Pre-tax margin TTM	0.0150
		g2,6	Pre-tax margin 5YA	0.0200
		g2,7	Net margin TTM	0.0200
		g2,8	Net margin 5YA	0.0250
g3	Market value ratios	g3,1	Earnings per share TTM	0.0250
		g3,2	Basics EPS ANN	0.0250
		g3,3	Diluted EPS ANN	0.0200
		g3,4	Book value per share MRQ	0.0200
		g3,5	Tangible book value per share MRQ	0.0200
		g3,6	Cash per share MRQ	0.0200
		g3,7	Cash flow per share TTM	0.0250
	Profitability ratios	g4,1	Return on equity TTM	0.0250
		g4,2	Return on equity 5YA	0.0300
		g4,3	Return on assets TTM	0.0300
		g4,4	Return on assets 5YA	0.0250
		g4,5	Return on equity TTM	0.0250
		g4,6	Return on investment 5YA	0.0300
		g4,7	ESP (MRQ) vs previous year quarter MRQ	0.0200
		g4,8	EPS (TTM) vs previous year TTM	0.0250
		g4,9	Sales (TTM) vs previous year TTM	0.0250
		g4,10	Sales (MRQ) vs previous year quarter MRQ	0.0300
g5	Liquidity ratios	g5,1	EPS growth in 5 years 5YA	0.0350
		g5,2	Sales growth in 5 years 5YA	0.0100
		g5,3	Capital expenditure growth in 5 years 5YA	0.0100
		g5,4	Acid-test ratio MRQ	0.0100
		g5,5	Solvency ratio MRQ	0.0100
		g5,6	Long-term debt to equity MRQ	0.0100
		g5,7	Total debt to equity MRQ	0.0100
g6	Effectiveness ratio	g6,1	Assest turnover TTM	0.0100
		g6,2	Inventory turnover TTM	0.0100
		g6,3	Profit per employee TTM	0.0100
		g6,4	Net income per employee TTM	0.0100
		g6,5	Accounts receivable turnover TTM	0.0100
g7	Dividend ratio	g7,1	Dividend yield ANN	0.0300
		g7,2	Average dividend yield over 5 years 5YA	0.0400
		g7,3	Dividend growth rate ANN	0.0400
		g7,4	Payout ratio TTM	0.0250

Source: Own elaboration.



The hierarchical ELECTRE III and distillation methods described in Section 3.2 were applied to solve each subproblem g_i (macro-criterion), and the integrated level. Table 3 presents the overall ranking, which assigns 29 positions to the analyzed companies' stocks. Microsoft (A17) ranks first and retains this position in the final ranking. Dupont (A8) and Raytheon Technologies (A26) share the 28th position, while Visa (A28) ranks second, and American Express (A2) is in 17th placeboth companies belonging to the same economic sector. In the last position are Boeing (A20) and Exxon Mobil (A9), which can be explained by the fact that during the COVID-19 pandemic, the air transportation sector was among the most affected due to the economic shutdown, business closures, and reduced population mobility. Although some companies share positions, the overall ranking (g₀), assigns Microsoft (A17), Visa (A28), Home Depot (A21), Intel (A11), and Goldman Sachs (A10) the top five spots as the best-performing stockshighlighting their status as technology and service companies. Table 4 presents the individual ranking,

Table 4. Individual ranking of company stocks.

Position g1 g5 g6 g2 **g**3 g4 **g**7 A28 A10 A17, A18 A21 A17 A17 A21 1 A28 A17 A25 A1, A17 A10 A30 A19 2 3 A6 A10 A23 A22 A18 A25 A14 A6 A15 A11 A14 A18 A11 A6, A15 4 A29 A6, A7 A29 A11 A4 A21 A4 5 A13, 6 A2 A13 A4 A16 A15 A28 A28 7 A7 A15 A21 A6 A26 A20 A12 8 A29 A16 A2 A10 Aı A24 Aı 0 A21 A19 A12 A27 A25 A11, A22 A27 10 A25 A22 A5 A7 A28 A13 A13 11 A11 A27 A17 A25 A13 A8 A10 A19 A2 Aı A12, A19 A30 A18 A7 12 A22 Aı A11 A29 A19 A27 A25 13 14 A13 A12 A30 A15 A24 A26 A16 A18 A13 A19 A18 A14 A16 A29 15 Aı A3 A28 A4, A30 A6 A9 A3 16 A8 A21 A22 A16 A11, A17 17 A23 A27 18 A23 A4 A18 A2 A23 A5 A5 A24 A5 A6 A14 A22 A3 A9 19 A16 A24 A24 A26 A3, A15 A23 A22 20 21 A12 A14 A27 A24 A4 Aı A30 22 A5 A29 A16 A3, A8 A2 A28 A2 A10 A23 A8, A26 23 A5 A12 A4 A23 A9 A30 A15 A9 A7 A7 A29 24 25 A27 A25 A9 A20 A14 A14 A26 26 A3, A30 A8, A9 A7 A8 A12 A8 A20 A26 A19 A20 A2 A24 27 28 A26 A20 A3 A5, 9 A21 A20 A20 A10 29

Source: Own elaboration.



where macro-criteria are analyzed based on their relative importance to the decision-maker.

Table 3. Overall ranking (g_0) of the Dow Jones Index

Position	g _o	Position	g _o
1	A17	16	A4
2	A28	17	A2
3	A21	18	A16
4	A11	19	A27
5	A10	20	A14
6	A6	21	A29
7	A18	22	A23
8	A25	23	A30
9	A22	24	A5
10	Aı	25	A24
11	A19	26	A3
12	A13	27	A9
13	A12	28	A8, A26
14	A15	29	A20
15	A7		

Source: Own elaboration.

Each macro-criterion is evaluated through a subset of sub-criteria (elementary criteria belonging to the lowest level of the hierarchy). Table 4 presents the rankings of each macro-criterion (g1...g10). The resulting rankings emerge from the interaction of elementary criteria that evaluate the corresponding macro-criteria. For the stock selection problem, the interaction of elementary criteria subsets was analyzed at the macro-criteria level (Level 2 of the hierarchy), and subsequently, the interaction of macro-criteria at the top level (Level 1) was considered to form a comprehensive stock selection model.

The relative importance of the marco-criteria is as follows: $g4 \succ g3 \succ g2 \succcurlyeq g1 \succ g7 \succ g5 \succ g6$, with the corresponding weights 0.2650, 0.1500, 0.1500, 0.1350, 0.0950 y 0.0400. In terms of profitability ratios (g4) the top positions are occupied by A21 > A1 = A17 > A22. Market value ratios (g3) rank A10 > A25 > A23 > A14 > A29. Results ratios (g2) show A28 > A17 > A10 > A11 > A6 = A7 ; and market ratios (g1) show A17 = A18 > A28 > A6 > A15.

Based on the multicriteria ranking, in the macrocriterion of market ratios (g1)there is a tie for first place between Microsoft (A17) and Visa (A28), followed by Home Depot (A21). Although these companies belong to different economic sectorsinformation services and financial services, respectively-they exhibit superior performance in financial indicators related to market value. The third position is held by Home Depot, a company in the construction and materials sector, as reflected in the overall ranking (g₀). Therefore, each of the seven rankings allows for identifying a stock's position within its respective group. To determine the hierarchical ranking, weights were established according to the decision-maker's judgment and investor profile regarding risk tolerance, which may influence the resulting ranking (see Table 2).

Given that Microsoft (A17) and Visa (A28) appear in top positions in the performance-related macro-criteria of the individual rankings (Table 4), Microsoft stands out in sub-criteria g_1 , g_5 , and g_6 , while Visa excels in g_2 , and Home Depot in g_4 . Although the profitability ratio (g_4) holds the highest weight (0.265), Home Depot performs lower in other macro-criteria, ranking 17th in g_2 , 28th in g_6 , and 9th in g_1 . Nevertheless, its strong performance in profitability ratios places it in the 3rd position of the overall ranking (g_0) . In terms of market value ratio (g_3) , the stock of Goldman Sachs (A10), a financial sector company, ranks first in the individual ranking and fifth in the overall ranking (g_0) .

The stocks positioned at the bottom include: 29th place, Boeing (A2o); 24th place, Exxon Mobil (A9); 23rd place, Chevron Corporation (A5); 22nd place, AT&T (A3), and Dupont (A8). Specifically, the stocks of Boeing (A2o), Technologies (A28), Dupont (A28), and Raytheon (A26) show low performance evaluations within the Dow Jones index. Boeing (A2o) ranks among the lowest across five macrocriteria (g2, g3, g4, g5, and g7). This is particularly attributable to its sector—aviation—which has been heavily affected by global market conditions due to the economic and financial consequences of the COVID-19 pandemic.

These variations are important to consider as they show how rankings may shift depending on the parameters applied to the same data. In this regard, rankings are not absolute; preferences and many other factors may vary depending on different quantitative parameters. Therefore, it is crucial to utilize methodologies adaptable to the decisionmaker's reality for investment portfolio integration based on companies' financial indicators and the investor's profile and preferences.

5. Conclusions

This study analyzes the performance of companies listed in the Dow Jones Index and evaluates the variables affecting stock performance using seven macro-criteria and 47 elementary criteria. From a methodological perspective, a Hierarchical Multicriteria Process (MCHP) was employed to analyze the performance of NYSE-listed companies. Subgroups of elementary criteria were assessed to understand their interaction and impact on the higher-level macro-criteria. This analytical process produced a preferential model, generating individual rankings for each macro-criterion and an overall ranking for the stock selection problem, taking into account the effects of COVID-19 on financial ratios.

MCHP allows for the evaluation of interactions between sub-criteria at all levels of the hierarchy to determine their influence across the structure. In the



context of stock selection, this approach highlights business opportunities and needs, enabling more robust and reliable decision-making. Applying MCHP to evaluate Dow Jones stocks can serve as a valuable tool for formulating more assertive policies and decisions within organizations. Consequently, this would promote favorable conditions for investors. In this regard, the ELECTRE III method provides decision-making support for real-world problems using a non-compensatory approach.

However, the research presents a limitation in that it does not consider stock volatility in the analysis. This limitation could be addressed by incorporating the beta coefficient as a criterion to evaluate volatility.

For future research, stock selection could support market portfolio integration using the Markowitz model and the Capital Asset Pricing Model (CAPM) proposed by Sharpe (1964). Additionally, other areas within the social sciences and economic phenomena could be explored to help minimize uncertainty in decision-making processes within public or private organizations.

References

- Aldalou, E., & Perçin, S. (2018). Financial Performance Evaluation of Turkish Airline Companies Using Integrated Fuzzy AHP Fuzzy Topsis Model. Uluslararası İktisadi ve İdari İncelemeler Dergisi.
- Albadvi, A., Chaharsooghi, S., & Esfahanipour, A. (2006). Decision making in stock trading: An application of PROMETHEE. European Journal of Operational Research, 177(2), 673–683.
- Almeida J., Figueira, J. R., y Roy, B. (2006). The software ELECTRE III-IV: Methodology and user Manual, Paris, Francia: University Paris-Dauphine Lamsade.
- Altınırmak, S., Gülcan2, B., & Çağlar, K. (2016). Analyzing securities investment trusts traded in BIST via AHP PROMETHEE methodology. Journal of International Scientific Publications, 10, 458–472.
- Angilella, S., Catalfo, P., Corrente, S., Giarlotta, A., Greco, S., & Rizzo, M. (2018). Robust sustainable development assessment with composite indices aggregating interacting dimensions: The hierarchical-SMAA-Choquet integral approach. Knowledge-Based Systems, 158, 136–153.
- Ariza, M., & Cadena, J. (2017). Selección de portafolios de renta variable: una propuesta a través de betas al alza y a la baja en el mercado colombiano. Criterio Libre. 11(19), 225-243.

- Bahloul, S., & Abid, F. (2013). A combined analytic hierarchy process and goal programming approach to international portfolio selection in the presence of investment barriers. International Journal of Multicriteria Decision Making, 3(1), 1–20. https://doi. org/10.1504/IJMCDM.2013.052455
- Basilio, M., De Freitas, J., Kämpffe, M. G., & Rego, R. (2018). Investment portfolio formation via multicriteria decision aid: A Brazilian stock market study. Journal of Modelling in Management, 13(12), 394–417. https:// doi.org/https://doi.org/10.1108/JM2-02-2017-0021
- Bay, Y., Yudan, W., & Li Quian. (2017). an optimal tradeoff model for portfolio selection with sensitivity of PARAMETERS Yanqin Bai , Yudan Wei and Qian Li. Journal of Industria l and Management Optimization, 13(2), 947–965. https://doi.org/10.3934/jimo.2016055
- Bodie, Z. (2019). Merton and the Science of Finance. Annual Review of Financial Economics, 11(1), 1–20.
- Bodie, Z., & Merton, R. (2003). Finanzas. Pearson Eduación.
- Boonjing, V., & Boongasame, L. (2016). Combinatorial Portfolio Selection with the ELECTRE III method: Case study of the Stock Exchange of Thailand (SET). Proceedings of the 2016 Federated Conference on Computer Science and Information Systems, 8(4), 719–724.
- Cao, L. J., & Tay, F. E. H. (2003). Support vector machine with adaptive parameters in financial time series forecasting. IEEE Transactions on Neural Networks, 14(6), .1506–1518.
- Cervelló, R., Guijarro, F., & Michniuk, K. (2014). Estrategia de inversión bursátil y reconocimiento gráfico de patrones: Aplicación sobre datos intradía del índice Dow Jones. Cuadernos de Administración.
- Corrente, S., Figueira, J. R., Greco, S., & Słowiński, R. (2017). A robust ranking method extending ELECTRE III to hierarchy of interacting criteria. Omega (United Kingdom), 73, 1–17.
- Corrente, S., Greco, S., & Słowiński, R. (2012). Multiple criteria hierarchy process in robust ordinal regression. Decision Support Systems, 53(3), 660–674. https:// doi.org/10.1016/j.dss.2012.03.004
- Creamer, G. (2012). Model calibration and automated trading agent for euro futures. Quantitative Finance, 12(4), 531-545.
- Creamer, G., & Freund, Y. (2007). A boosting approach for automated. Journal of Trading, 2(3), 84–96.
- Chahuán, K. (2018). Relación Dow Jones sustainability index Chile e ingresos, resultados y rentabilidad sobre patrimonio de empresas. Capic Review, 16. https:// doi.org/10.35928/cr.vol16.2018.68
- Dempster, M. A. H., Payne, T. W., Romahi, Y., & Thompson, G. W. T. (2001). Computational learning techniques for intraday FX trading using popular

technical indicators. EEE Transactions on Neural Networks, 12(4), 744–754.

- Ehrgott, M., Klamroth, K., & Schwehm, C. (2004). An MCDM approach to portfolio optimization. 155, 752–770. https://doi.org/10.1016/S0377-2217(02)00881-0
- Elselmy, H., Ghoneim, A., & Elkhodary, I. (2019). Portfolio selection factors: Egypt equity market case study. ACM International Conference Proceeding Series, 212–216.
- Giannoulis, C. & Ishizaka, A. (2010). A web-based decision support system with ELECTREIII for a personalized ranking of British universities," Decision Support Systems, 48(3), 488-497.
- Govindan, K., & Jepsen, M. B. (2016). ELECTRE: A comprehensive literature review on methodologies and applications. European Journal of Operational Research, 250(1), 1–29.
- Guerrero-Baena, D. D., Gómez-Limón, J. A., & Fruet Cardozo, V. V. (2014). Are multi-criteria decision making techniques useful for solving corporate finance problems? A bibliometric analysis. Revista de Metodos Cuantitativos Para La Economia y La Empresa, 17(1), 60–79.
- Kimoto, T., Asakawa, K., Yoda, M., & Takeoka, M. (1993). Stock market prediction system with modular neural networks. Neural Networks in Finance and Investing, 343-357.
- Lima, A., & Soares, V. (2013). Financial ratios applied to portfolio selection : Electre III methodology in buyand-hold strategy Indicadores financeiros aplicados à seleção de carteiras : Metodologia Electre III numa estratégia de buy-and-hold. Organizações Em Contexto, 9(17), 281–319.
- Lopez-Dumrauf, G. (2003). Finanzas corporativas. Buenos Aires: Grupo Guia.
- Lu, C. J., Lee, T. S., & Chiu, C. C. (2009). Financial time series forecasting using independent component analysis and support vector regression. Decision Support Systems, 47(2), 115–125.
- Macharis, C., Brans, J. P., Mareschal, B., (1998) The GDSS ROMETHEE procedure: a PROMETHEE–GAIA based procedure for group decision support, Journal of Decision Systems, 7, pp. 283–307.
- Mahfoud, S., & Mani, G. (1996). Financial forecasting using genetic algorithms. Applied Artificial Intelligence, 10(6), 543–565.
- Mandziuk, J., & Jaruszewicz, M. (2011). Neuro-genetic system for stock index prediction. Journal of Intelligent & Fuzzy Systems, 22(2–3), 93–123.
- Milanesi, G. (2016). Un modelo de opciones barreras para estimar las probabilidades de fracasos financieros de empresas. Barrier options model for estimate firm's probabilities for financial distress. TEC Empresarial. https://doi.org/10.18845/te.v10i3.2936

- Mansour, N., Cherif, M. S., & Abdelfattah, W. (2019). Multi-objective imprecise programming for financial portfolio selection with fuzzy returns. Expert Systems With Applications.
- Mohammad, J., Mohammad, E., & Sanam, B. (2012). Selection of Portfolio by using Multi Attributed Decision Making. American Journal of Scientific Research, 1450-223X(44), 15–29.
- Moody, J., & Saffell, M. (2001). Learning to trade via direct reinforcement. IEEE Transactions on Neural Networks, 12(4), 875–889.
- Moody, J., Wu, L., Liao, Y., & Saffell, M. (1998). Performance functions and reinforcement learning for trading systems and portfolios. Journal of Forecasting, 17(5), 441–471.
- OJ., L., J.W., & Zhang, B. T. (2002). Stock trading system using reinforcement learning with cooperative agents. Proceedings of the 19th International Conference on Machine Learning, 451–458.
- Pätäri, E., Karell, V., Luukka, P., & Yeomans, J. S. (2017). Comparison of the multicriteria decisionmaking methods for equity portfolio selection: The U.S. evidence. European Journal of Operational Research, 265(2), 655–672. https://doi.org/10.1016/j. ejor.2017.08.001
- Shabani Vezmelai, A., Lashgari, Z., & Keyghobadi, A. (2015). Portfolio selection using ELECTRE III: Evidence from Tehran Stock Exchange. Decision Science Letters, 4(2), 227–236.
- Shah, P., Mallory, M. L., Ando, A. W., & Guntenspergen, G. R. (2017). Fine-resolution conservation planning with limited climate-change information. Conservation Biology, 31(2).
- Sharpe, W. (1964). Capital Asset Prices. The Journal of Finance, 19(3).
- Spronk, J., Steuer, R. E., & Zopounidis, C. (2016). Multicrieria Decision Aid/Analysis in Finance. In International Series In operations Research & Management Science (pp. 1011–1065).
- Suárez, L., Pimiento, N., & Duarte, J. (2018). Selección de portafolios de inversión socialmente responsables usando el método de las restricciones y la técnica multicriterio Proceso Analítico Jerárquico. Revista EIA. https://doi.org/10.24050/reia.voio.634
- Sun, Y. F., Grace, A., Teo, K. L., & Zhou, G. L. (2015). Portfolio optimization using a new probabilistic risk measure. Journal of Industrial and Management Optimization, 11, 1275–1283.
- Sun, X., Zheng, X. & Li, D. (2013). Recent advances in mathematical programming with semicontinuous variables and cardinality constraint. Journal Operations Research Society of China, 1, 55–77.
- Tay, F. E. H., & Cao, L. J. (2002). Modified support vector machines in financial time series forecasting.



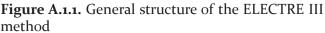
Neurocomputing, 48(1-4), 559-565.

- Teo, K. & Yang, X. (2001). Portfolio selection problem with minimax type risk function. Annals of Operations Research, 101, 333–349.
- Tsang, E., Yung, P., & Li, J. (2004). 'EDDIE-automation', A Decision Support Tool for Financial Forecasting. Decision Support Systems, Periodical Style, 37, 559– 565.
- Tian, Y., Fang, S., Deng, Z. & Jin, Q. (2016). Cardinality constrained portfolio selection problem: A completely positive programming approach. Journal of Industrial and Management Optimization, 12, 1041–1056.
- Useche, A. J. (2015). Construcción de portafolios de inversión desde las finanzas del comportamiento: una revisión crítica. Cuadernos de Administración. https://doi.org/10.11144/javeriana.cao28-51.cpif
- Zhu, S.S., Li, D., & Sun, X. L. (2010). Portfolio selection with marginal risk control. The Journal of Computational Finance, 14(1), 3–28. https://doi.org/doi:10.21314/ JCF.2010.213

Apendix

A.1 Illustration of the Application of the ELECTRE III Method

This section presents an illustrative example of the application of the ELECTRE III method. Figure A.1.1 displays a general overview of the method's application process. For the purposes of this illustration, data from a problem presented in Macharis, Brans, and Mareschal (1998) are used. A detailed explanation of the method can be found in Almeida, Figueira, and Roy (2006). Table A.1.1 presents the evaluation matrix, while Table A.1.2 provides the parameters that will be used in the application of the ELECTRE III method.



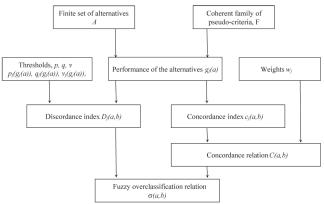


Table A.1.1. Evaluation matrix of alternatives

Code	Country	g1	g2	g3	g 4
Aı	Italy	8	0.5	9	0
A2	Belgium	1	4	3	5
A3	Germany	4	3.5	7	65
A4	Switzerland	7	0	10	0
A5	Austria	3	4.5	2	10
A6	France	5	3.5	4	10

Table A.1.2 Parameters of the ELECTRE III Method

	g1	g2	g3	g4
	Min	Min	Мах	Min
W	0.589	0.178	0.120	0.113
q	3.193	1.372	0.196	3.893
p	3.690	1.698	2.127	41.031
v	-	2.937	-	-

Concordance index

$$\phi_{t}(a,b) = \begin{cases} 1 & \text{if} \quad g_{t}(b) - g_{t}(a) \leq q_{t}, (aS_{t}b) \\ \frac{p_{t} - [g_{t}(b) - g_{t}(a)]}{p_{t} - q_{t}} & \text{if} \quad q_{t} < g_{t}(b) - g_{t}(a) < p_{t}, (bQ_{t}a) \\ 0 & \text{if} \quad g_{t}(b) - g_{t}(a) \geq p_{t}, (bP_{t}a) \end{cases}$$
(A.1)

The concordance index between the alternatives Italy (a) and Switzerland (b), considering criterion g3, is calculated using Equation A.1 as follows.

Given $\varphi_t(a, b) \rightarrow \varphi_t$ (Italy, Switzerland), the following values are available for criterion 3, g₃ (Italy) = 9, g₃ (Switzerland) = 10. The difference between both cities for criterion g₃ is g₃ (Switzerland) - g₃ (Italy) = 1. This difference is neither less than or equal to q₃, (q₃ = 0.196), nor greater than p₃, (p₃ = 2.127). Therefore, the calculation corresponds to the second case of Equation A.1:

$$\frac{p_3 - [g_3(b) - g_3(a)]}{p_3 - q_3} = \frac{2.127 - [1]}{2.127 - 0.196} = 0.58$$

The concordance indices resulting from the comparison of each country with the remaining countries are presented in Table A.1.3.

Soucre: Almeida, Figueira & Roy (2006).

Table A.1.3. Concordance indices

		Italy	(A1)			Swit	zerland (A	44)	
	g1	g2	g3	g4		gı	g2	g3	g4
(A1,A2)	0	1	1	1	(A4,A1)	1	1	1	1
(A1,A3)	0	1	1	1	(A4,A2)	0	1	1	1
(A1,A4)	1	1	0.58	1	(A4,A3)	1	1	1	1
(A1,A5)	0	1	1	1	(A4,A5)	0	1	1	1
(A1,A6)	1	1	1	1	(A4,A6)	1	1	1	1
	В	elgium (A	12)			А	ustria (A5	;)	
(A2,A1)	1	0	0	0.97	(A5,A1)	1	0	0	0.84
(A2,A3)	1	1	0	1	(A5,A2)	1	1	0.58	0.97
(A2,A4)	1	о	0	0.97	(A5,A3)	1	1	о	1
(A2,A5)	1	1	1	1	(A5,A4)	1	о	о	0.84
(A2,A6)	1	1	0.58	1	(A5,A6)	1	1	0.066	1
	Ge	ermany (A	3)			F	rance (A6)	
(A3,A1)	1	0	0.066	0	(A6,A1)	1	0	о	0.84
(A3,A2)	1	1	1	0	(A6,A2)	0	1	1	0.97
(A3,A4)	1	0	0	0	(A6,A3)	1	1	о	1
(A3,A5)	1	1	1	0	(A6,A4)	1	0	о	0.84
(A3,A6)	1	1	1	0	(A6,A5)	1	1	1	1

Discordance index

$$d_{t}(a,b) = \begin{cases} 1, & \text{if} \quad g_{t}(b) - g_{t}(a) \ge v_{t}, \\ \frac{[g_{t}(b) - g_{t}(a)] - p_{t}}{v_{t} - p_{t}} & \text{if} \quad p_{t} < g_{t}(b) - g_{t}(a) < v_{t}, \\ 0, & \text{if} \quad g_{t}(b) - g_{t}(a) \le p_{t}. \end{cases}$$
(A.2)

The discordance index between the alternatives Belgium (a) and Italy (b), considering criterion g_2 , is calculated using Equation A.2 as follows.

Given $d_t(a,b) \rightarrow d_t$ (Belgium, Italy), the following values are available for criterion 2,g2 (Belgium) = 4, g2 (Italy)= 0.5. The difference between both cities for criterion g2 is $g_2(a)-g_2(b)=3.5$. This difference is greater than v_2 , ($v_2 = 2.937$). Therefore, the first case of Equation A.2 applies. The complete discordance index data are shown in Table A.1.4. $g_t(a)-g_t(b) \geq v_2$, then $d_t(a,b)=1$

Full concordance index

$$C_r(a,b) = \frac{\sum_{t \in E(g_r)} w_t \varphi_t(a,b)}{\sum_{t \in E(g_r)} w_t}$$
(A.3)

The comprehensive concordance index corresponds to the weighted sum of each concordance index value (c_t , obtained through Equation A.1) by its corresponding importance weight (w_t).

$$C(a,b) = w_{1} * c_{1}(a,b) + \dots + w_{ll} * c_{ll}(a,b)$$

Equation A.3 represents this weighted sum. An example of its application is the calculation of the comprehensive concordance index between Italy and Belgium, as follows.

$$C(A_1, A_2) \rightarrow C(Italy, Belgium)$$

C (Italy, Belgium) = $0.589^{*}0 + 0.178^{*}1 + 0.12^{*}1 + 0.113^{*}1 = 0.41$

C (Italy, Belgium) =
$$0.41$$



Table A.1.4. Discordance indices

		Italy (A1)			Switzerland (A4)							
	g1	g2	g3	g 4		g1	g2	g 3	g4			
dj(A1,A2)	0	0	0	0	dj(A4,A1)	0	0	0	0			
dj(A1,A3)	0	0	0	0	dj(A4,A2)	о	0	0	0			
dj(A1,A4)	0	0	0	0	dj(A4,A3)	0	0	0	0			
dj(A1,A5)	0	0	0	0	dj(A4,A5)	0	0	0	0			
dj(A1,A6)	0	0	0	0	dj(A4,A6)	0	0	0	0			
		Belgium (A2)					Austria (A5)					
dj(A2,A1)	0	1	0	0	dj(A5,A1)	0	1	0	0			
dj(A2,A3)	0	0	0	0	dj(A5,A2)	0	0	0	0			
dj(A2,A4)	0	1	0	0	dj(A5,A3)	0	0	0	0			
dj(A2,A5)	0	0	0	0	dj(A5,A4)	0	1	0	0			
dj(A2,A6)	0	0	0	0	dj(A5,A6)	0	0	0	0			
	(Germany (A3)					France (A6)					
dj(A3,A1)	0	1	0	0	dj(A6,A1)	0	1	0	0			
dj(A3,A2)	0	0	0	0	dj(A6,A2)	0	0	0	0			
dj(A3,A4)	0	1	о	о	dj(A6,A3)	0	0	0	0			
dj(A3,A5)	0	0	0	о	dj(A6,A4)	0	1	0	0			
dj(A3,A6)	0	0	0	0	dj(A6,A5)	0	0	0	0			

The complete data for the comprehensive concordance index are shown in Table A.1.5.

	Aı	A2	A3	A4	A5	A6
Aı	1	0.41	0.41	0.95	0.41	1
A2	0.7	1	0.88	0.7	1	0.95
A3	0.6	0.89	1	0.59	0.89	0.89
A4	1	0.41	1	1	0.41	1
A5	0.68	0.95	0.88	0.68	1	0.89
A6	0.68	0.41	0.88	0.68	1	1

Credibility index

$$\sigma_r(a,b) = \begin{cases} C(a,b) x \prod_{g_t \in E(g_r)} \frac{1 - d_t(a,b)}{1 - C_r(a,b)} & \text{if } d_t(a,b) > C_r(a,b) \end{cases}$$
(A.4)

The credibility index corresponds to reducing its value (credibility) for pairs of alternatives where $d_t(a, b) > C(a, b)$. Some examples of this are the pairs

 $d_2(A2, A1)$ and $d_2(A2, A4)$ (see Table A.1.4). Table A.1.6 presents the credibility index, where it can be observed how the complete concordance index is reduced to o for the pairs (A2, A1) and (A2, A4) due to the discordance present in these alternative pairs.

	Aı	A2	A3	A4	A5	A6
Aı	1	0.41	0.41	0.95	0.41	1
A2	0.7	1	0.88	0.7	1	0.95
A3	0.6	0.89	1	0.59	0.89	0.89
A4	1	0.41	1	1	0.41	1
A5	0.68	0.95	0.88	0.68	1	0.89
A6	0.68	0.41	0.88	0.68	1	1

A.2 Financial ratios data of companies in the Dow Jones Index

Table A.2.1 Performance of financial ratios of companies in the Dow Jones Index (Part 1, continued...)

		M	arket ra	tios (gı))				1	Results r	atios (g2)						Marke	t value	ratios (g	g3)	
	g1,1	g1,2	g1,3	g1,4	g1,5	g1,6	g2,1	g2,2	g2,3	g2,4	g2,5	g2,6	g2,7	g2,8	g3,1	g3,2	g3,3	g3,4	g3,5	g3,6	g3,7
Aı	21.4	3.56	35.5	35.5	8.87	о	0.4876	0.4875	0.2222	0.2223	0.2085	0.2142	0.1676	0.165	55.28	9.32	9.25	22.3	-12	8.72	12.55
A2	39.4	3.22	45.2	45.2	5.2	5.19	0.7242	0.7084	0.116	0.1839	0.116	0.1839	0.0847	0.1369	45.94	3.77	3.77	28.6	28.6	40	5.71
A ₃	0	1.24	17	17	1.31	о	0.5342	0.5305	0.0373	0.1245	-0.0166	0.089	-0.0222	0.0628	23.93	-0.8	-0.8	22.7	-17	1.37	3.41
A4	43.7	3.04	64.4	64.4	8.27	16.6	0.3044	0.3118	0.1091	0.1146	0.0957	0.1018	0.0697	0.0763	76.15	5.36	5.32	28.1	14	17.15	9.74
A5	0	2.12	0	0	1.51	1.57	0.4642	0.436	-0.0599	0.022	-0.0791	0.0403	-0.059	0.0297	50.37	-3	-3	68.4	66.1	2.92	7.45
A6	21.6	4.54	25.3	26.3	5.57	74.6	0.6417	0.6301	0.2593	0.2614	0.2641	0.2695	0.2109	0.2202	11.32	2.65	2.64	9.27	0.69	7.25	2.81
A7	29.8	6.96	142	68	11.9	0	0.5931	0.6096	0.2901	0.2249	0.2953	0.2397	0.2353	0.1899	7.64	1.8	1.79	4.49	-2.2	2.54	2.15
A8	41.7	2.02	20.4	20.4	1.47	о	0.3371	0.2755	-0.1129	0.0061	-0.142	0.0009	-0.1409	0.0028	27.73	-4	-4	52.4	-3.9	3.46	0.3
A9	0	1.35	0	0	1.53	1.54	0.3032	0.3083	-0.1717	0.0128	-0.1617	0.0424	-0.1303	0.0297	41.81	-5.3	-5.3	37.1	37.1	1.03	5.32
A10	8.42	1.92	0	о	1.18	1.18	0.8103	0.6918	0.3213	0.2317	0.3213	0.2317	0.2489	0.1788	168.59	25	24.7	286	286	557.01	45.81
Au	13.2	3.4	17	17	3.26	5.88	0.5601	0.5974	0.3041	0.2923	0.3221	0.3084	0.2684	0.2567	18.4	4.98	4.94	20	11.1	5.88	7.83
A12	21.2	1.61	12.9	12.9	5.75	о	0.4832	0.4737	0.063	0.1281	0.063	0.1281	0.0762	0.126	82.11	6.3	6.26	23.1	-59	15.47	13.73
A13	29.7	5.11	43.5	43.5	6.67	0	0.657	0.6711	0.1998	0.2263	0.1998	0.2263	0.1745	0.1918	30.94	5.47	5.4	24	-10	9.57	8.11
A14	12.1	3.73	0	0	1.85	2.04	0	о	0.4044	0.3533	0.4044	0.3533	0.3282	0.2756	40.11	8.89	8.88	92.7	74.7	234.9	10.23
A15	36.7	8.98	198	198	0	о	0.5077	0.4813	0.3813	0.3891	0.3197	0.3424	0.2463	0.2509	25.61	6.35	6.31	-11	-14	4.63	8.64
A16	27.6	4.04	0	0	7.66	о	0.681	0.6956	0.1647	0.1782	0.1832	0.1849	0.1476	0.1495	18.91	2.79	2.78	10	-3.8	3.19	4.22
A17	38.7	12.8	65	56.6	15	24.6	0.6835	0.6571	0.3918	0.3272	0.3962	0.3329	0.3347	0.2822	20.05	5.82	5.76	17.3	10.5	17.49	8.28
A18	63.1	5.48	0	93.9	17.7	18.5	0.4336	0.4451	0.1072	0.1179	0.104	0.1198	0.089	0.1032	24.16	1.63	1.6	7.56	7.23	7.93	2.69
A19	30.1	5	66.1	66.1	3.31	о	о	0.7895	0.1789	0.2035	0.1789	0.1886	0.1675	0.1745	7.55	1.26	1.24	11.4	-2.7	2.2	2.12
A20	0	2.5	0	0	0	о	-0.0884	0.1366	-0.2177	0.0327	-0.2468	0.0249	-0.2036	0.0234	103.16	-21	-21	-31	-49	43.94	-17.05
A21	27	2.63	35	35	105	о	0.3395	0.3411	0.1384	0.1424	0.1285	0.1327	0.0974	0.095	122.61	12	11.9	3.06	-3.6	7.33	14.28
A22	25.9	4.57	51.7	41	7.15	0	0.5191	0.5047	0.2378	0.1854	0.2316	0.1842	0.1884	0.1422	28.27	5.13	4.96	19.6	-6.9	4.85	6.35
A23	14.7	1.21	6.86	6.86	1.33	1.56	о	о	0.1116	0.1189	0.101	0.1071	0.0842	0.0867	126.04	10.6	10.5	116	98.7	2.86	13.71
A24	0	5.55	168	128	3.94	0	0.2985	0.4136	-0.0737	0.1815	-0.0711	0.1821	-0.0754	0.1276	33.52	-1.6	-1.6	47.2	-22	9.58	0.44
A25	22.2	1.4	23.7	20.1	5.32	5.32	0	о	0.0918	0.0788	0.0857	0.0725	0.0657	0.0532	273.87	16.2	16	73.3	73.3	24.24	21.05
A26	0	2.09	0	0	1.64	0	0.1578	0.2299	-0.0335	0.0877	-0.0416	0.0757	-0.0517	0.0552	38.52	-2.3	-2.3	47.5	-15	5.79	0.84
A27	13.4	1.86	21.3	21.3	3.52	о	0.6009	0.5852	0.2111	0.1973	0.1868	0.1678	0.143	0.1269	30.98	4.3	4.3	16.4	-15	5.36	8.47
A28	52.1	22.5	68.7	72.4	13.2	0	0.7904	0.8124	0.6476	0.6277	0.6315	0.6188	0.4991	0.4648	8.34	5.27	4.33	19	-3.5	9.09	4.47
A29	29.6	0.71	20.1	20.1	4.89	7.6	0.2483	0.2511	0.0253	0.0356	0.0368	0.034	0.0245	0.0237	196.64	4.77	4.75	28.7	18.4	6.29	8.74
A30	0	0.35	19.6	18.6	2.2	0	0.1995	0.2299	-0.0053	0.0375	-0.0067	0.033	-0.0067	0.0271	153.89	0.52	0.52	24.4	-2.1	1.19	1.2



Table A.2.1 Performance of financial ratios of companies in the Dow Jones Index (Part 2, continued...)

	Profability ratios (g4)										Liquidity ratios (g5)						
	g4,1	g4,2	g4,3	g4,4	g4,5	g4,6	g4,7	g4,8	g4,9	g4,10	g5,1	g5,2	g5,3	g5,4	g5,5	g5,6	g5,7
Aı	0.4696	0.4854	0.1172	0.1364	0.1444	0.1702	0.4302	0.1833	0.0015	0.0582	0.0404	0.0123	0.0054	1.35	1.89	1.3981	1.4607
A2	0.1318	0.2497	0.0161	0.0299	0.0315	0.0563	-0.1392	-0.5286	-0.1792	-0.1888	-0.0569	0.0215	0.0196	0	0	1.8688	5.7303
A ₃	-0.031	0.0637	-0.0071	0.0222	-0.0084	0.0262	-6.9751	-1.3994	-0.0521	-0.0241	о	0.0319	-0.0399	0	0.82	0.9511	0.9726
A4	0.1951	0.2543	0.0371	0.0462	0.0558	0.0707	-0.3548	-0.4889	-0.224	-0.1452	0.0495	-0.0235	-0.083	1.09	1.53	1.6958	2.424
A5	-0.0402	0.0257	-0.0233	0.0151	-0.0261	0.017	0.9075	-3.0188	-0.3277	-0.2836	о	-0.0613	-0.2127	0.92	1.18	0.3248	0.3365
A6	0.2714	0.2137	0.1089	0.0971	0.1482	0.1279	-0.1103	-0.1329	-0.0684	-0.0037	0.086	0.0006	-0.089	1.56	1.61	0.2442	0.372
A7	0.4048	0.3501	0.0895	0.08	0.1214	0.1172	-0.288	-0.1333	-0.1141	-0.0504	0.0144	-0.0571	-0.1435	1.09	1.32	2.0791	2.2174
A8	-0.073	-0.0007	-0.041	0.0005	-0.0456	0.0007	1.3966	-2.9164	-0.0518	0.0092	О	-0.16	-0.2045	1.52	2.31	0.5663	0.5665
A9	-0.1288	0.0369	-0.0669	0.0198	-0.083	0.0243	-5.0558	-2.6506	-0.3013	-0.2743	0	-0.0573	-0.0819	0.46	o.8	0.3002	0.4304
A10	0.1619	0.1076	0.0126	0.0093	0.0297	0.0205	4.9793	1.1844	0.1287	0.5826	0.153	0.0742	0.2804	0	0	2.2347	7.7143
A11	0.2634	0.2437	0.1443	0.1401	0.1723	0.1656	-0.1003	0.0456	0.082	-0.0114	0.1618	0.0706	0.1388	1.57	1.91	0.4183	0.4492
A12	0.2709	0.5397	0.0364	0.0753	0.0487	0.1057	-0.6608	-0.4166	-0.0457	-0.0647	-0.1438	-0.0207	-0.0489	0.94	0.98	2.6388	2.9876
A13	0.2349	0.2387	0.0867	0.0991	0.1134	0.1252	-0.5662	-0.0406	0.0064	0.0833	-0.003	0.0334	-0.0068	0.99	1.21	0.5157	0.5573
A14	0.1617	0.121	0.0119	0.0113	о	о	4.738	0.4223	-0.2711	-0.2552	0.0814	0.0483	о	0	0	0.9954	2.2743
A15	0	0	0.0945	0.144	0.1047	0.1586	-0.1393	-0.211	-0.1009	-0.0211	0.0563	-0.0545	-0.0199	1	1.01	о	0
A16	0.276	0.2004	0.0805	0.0726	0.1122	0.0947	-1.8803	-0.2795	0.0246	0.0545	0.1221	0.0397	0.2956	0.79	1.02	1.0017	1.2557
A17	0.427	0.3603	0.1748	0.1304	0.2232	0.1733	0.3405	0.1693	0.1418	0.1672	0.313	0.0885	0.2104	2.55	2.58	0.5064	0.5532
A18	0.3268	0.3439	0.1099	0.1579	0.1516	0.2172	0.684	-0.2161	-0.067	0.025	-0.0292	0.041	0.0243	2.02	2.78	0.7889	0.7892
A19	0.1105	0.124	0.0436	0.0481	0.0544	0.0596	1.2513	-0.3255	0.0179	0.1182	0.0224	-0.0302	0.1328	1.04	1.35	0.5872	0.6299
A20	0	0	-0.0836	0.0175	-0.2374	0.0497	-7.2164	-16.433	-0.3084	-0.2314	0	-0.094	-0.1186	0.46	1.39	0	0
A21	140.61	9.2595	0.2112	0.217	0.3205	0.333	0.1622	0.165	0.1985	0.2513	0.1692	0.0834	0.1038	0.51	1.23	10.858	11.288
A22	0.2955	0.176	0.1202	0.0788	0.1648	0.1055	0.0427	1.9623	0.063	0.0825	0.1183	0.0006	-0.0383	0.59	0.78	0.4673	0.6455
A23	0.097	0.106	0.0238	0.0248	O	O	0.5253	0.0566	0.0143	0.0458	-0.0067	0.0358	O	0	0	0.2209	0.2243
A24	-0.0567	0.1286	-0.0227	0.0623	-0.0292	0.081	-0.9864	-1.4594	-0.1918	-0.2217	0	0.045	-0.0117	1.26	1.31	0.629	0.6932
A25	0.2674	0.2351	0.0877	0.0796	0	0	0.4451	0.2318	0.0676	0.0896	0.2167	0.1036	0.0568	0	0	0.5399	0.6674
A26	-0.0546	0.0677	-0.0194	0.0239	-0.0272	0.0332	-0.8713	-1.5028	0.2478	0.4041	0	0.0017	-0.012	0.95	1.21	0.4299	0.441
A27	0.2755	0.3557	0.0603	0.061	0.0705	0.0711	-0.1006	-0.0764	-0.0271	-0.0024	-0.0032	-0.0051	-0.0602	1.33	1.38	1.8156	1.9024
A28	0.3247	0.3117	0.1381	0.1376	0.1712	0.1634	-0.1402	-0.1844	-0.087	-0.0606	0.1093	0.095	0.122	0	2.12	0.5588	0.5588
A29	0.1737	0.1533	0.0561	0.0563	0.0899	0.0897	-1.5099	-0.087	0.0672	0.0735	0.0077	0.0301	-0.0221	0.49	0.97	0.5566	0.6039
A30	-0.0359	0.1326	-0.0099	0.0494	-0.0151	0.0712	0.0879	-1.244	0.0293	0.046	-0.3357	0.0617	0.0189	0.56	0.83	0.5687	0.8144

Dividend ratio (g7)

g6,1 g6,2 g6,3 g6,4 g6,5 g7,1 g7,2 g7,3 g7,4 Aı 0.7 3.94 338.83 56.78 6.78 0.03 0.0274 0.0775 0.6293 A2 0 0 0.0116 0.0141 0.4586 0.19 0 0.74 0.095 0.0699 A3 746.78 -16.61 7.38 0.0562 0 0.32 0 0.02 A4 0.53 2.56 429.06 29.9 5.28 0.0177 0.0246 0.0995 0.77 A5 0.39 8.76 1.97 -116.49 7.6 0.0499 0.0413 0.061 0 0.0287 0.0285 A6 0.0963 0.6009 0.52 12.34 619.69 130.7 9.76 A_7 0.38 0.031 0.9096 4.04 411.13 96.74 9.28 0.0315 0.0374 A8 0.29 3.36 599.91 -84.53 5.52 0.0156 0 0.0172 0 A9 0.51 6.66 2.48 -323.28 7.51 0.0611 0.0458 0.0438 0 0.0928 A10 0.05 1.5 0.0148 0.0154 0.1991 0 372.4 0.42 0.2668 A11 0.54 704.04 188.96 10.8 0.0214 0.025 0.07 3.99 A12 0.48 196.17 14.96 0.0492 0.0441 0.0333 1.033 22 9.53 0.0623 A13 0.5 3.09 614.01 107.17 5.89 0.0252 0.0259 0.7271 A14 0 0 0 0 0 0.0237 0.0238 0.2084 0.2148 A15 0.38 186.69 96.04 23.65 8.86 0.0223 0.0242 0.0958 0.7933 A16 0.55 648.58 95.72 6.56 0.0339 0.03 0.0167 0.8925 2.49 A17 0.52 25.89 940.39 314.79 6.03 0.0086 0.015 0.0916 0.3152 A18 510.7 45.46 9.46 0.0082 1.23 3.49 0.0107 0.11 0.4504 A19 0.26 0 533.86 89.44 5.7 0.0415 0.0377 0.059 1.2271 A20 0.81 416 -84.69 -0.2874 0.41 5.27 0 0.0207 0 A21 2.17 5.6 261.71 25.49 51.8 0.0204 0.0213 0.1901 0.5014 A22 0.64 6.2 747.22 140.79 14.8 0.0254 0.0294 0.0392 0.5705 A23 0.28 88.14 0 1.05 0 0.022 0.0227 0.0631 0.323 A24 0.3 299.31 -22.56 0 0.013 -0.1737 0 27.94 3.9 0.0128 0.1888 A25 1.33 0 796.72 52.37 0 0.0139 0.2109 0.0366 A26 0.38 5.16 312.64 -16.18 0 3.49 0.0244 -0.074 A27 0.42 31.82 970.44 138.79 5.04 0.0435 0.042 0.0211 0.5777 A28 0.28 0 1.05 522.93 12.6 0.0057 0.006 0.2205 0.2603 A29 5.96 2.29 243.11 87.4 0.0157 0.0198 0.0192 0.4527 9.4 A30 1.48 11.76 599.08 -4.04 21.5 0.0348 0.0254 0.0685 0

Table A.2.1 Financial ratios performance of companies in the Dow Jones Index (Part 3)

Effectiveness ratio (g6)